



A-LEVEL

MATHEMATICS

A Level Maths Predicted Paper 3

2027 (June)

Statistics and Mechanics (Set 1)

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A/Prof (Dr) Keith



A Level Maths Predicted Paper 2027 (June) Paper 3: Statistics and Mechanics (Set 1)

Duration: 2 Hours

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This AL Maths paper 3 (Statistics and Mechanics: Predicted Paper June 2027: Set 1) has been created based on the most common topics from previous past papers. This paper should be excellent for helping students revise for exams; however, it should not be relied upon as the sole basis for revision

Grade Boundary: A: 60% and A*: 75%

Why Should You Practise Harder A-Level Mathematics Papers?

The June 2026 A-Level Mathematics Paper 1 was widely considered to be around **10–15% more challenging** than a standard A-Level Mathematics paper. To help our students prepare for any level of difficulty, we have designed our **June 2027 Predicted Papers** to be approximately **10–15% harder than a typical A-Level Mathematics examination**.

Five Advantages of Practising Harder A-Level Mathematics Papers

1. **Builds Confidence for the Real Examination**
 - If you can solve questions that are more demanding than those typically set in the examination, you will approach the actual exam with greater confidence and reduced anxiety.
2. **Develops Stronger Problem-Solving Skills**
 - More challenging questions require deeper mathematical thinking and encourage you to apply multiple topics together, improving your analytical and reasoning skills.
3. **Prepares You for Unexpected Questions**
 - Examination boards often introduce unfamiliar or more demanding questions. Practising harder papers helps you adapt quickly and tackle new question styles with confidence.
4. **Improves Speed, Accuracy and Time Management**
 - Working through more difficult papers under timed conditions enhances your efficiency, enabling you to complete standard examination questions more quickly and accurately.
5. **Increases the Chances of Achieving Top Grades**
 - Students who regularly practise challenging questions are generally better prepared for high-mark and extension questions, giving them a stronger opportunity to achieve A* grades.

Remember

"Train harder than the examination, and the examination will feel easier."

Consistent practice, careful review of mistakes, and perseverance are the keys to success in A-Level Mathematics.

Wishing all our students every success in the June 2027 A-Level Mathematics examinations!

A/Prof (Dr) Keith

1. A team of environmental economists is investigating whether wealthier countries tend to produce higher levels of carbon dioxide (CO₂) emissions per person. The study aims to understand whether economic prosperity is associated with increased environmental impact, which may help inform future climate and sustainability policies.

The researchers collect a simple random sample of 20 countries from around the world. For each country, they record:

- x : Gross Domestic Product (GDP) per capita (US dollars), representing the average annual income per person.
- y : Average annual CO₂ emissions per person (tonnes).

Using the original data, they calculate the product moment correlation coefficient to be

$$r = 0.41.$$

(a) State suitable null and alternative hypotheses and carry out a 5% level of significance hypothesis test to determine whether there is sufficient statistical evidence to conclude that the population correlation coefficient between GDP per capita and annual CO₂ emissions is positive.

You should state your conclusion clearly in the context of this investigation.

The researcher believes that the relationship between GDP per capita and CO₂ emissions may not be linear. Instead, she suspects that a power model would better describe the relationship. She therefore transforms the variables using

$$m = \log_{10} x \quad c = \log_{10} y.$$

Using the transformed data, the least-squares regression line is found to be

$$c = -2.05 + 0.76m.$$

The corresponding product moment correlation coefficient for the transformed variables is

$$r = 0.90.$$

(b) Explain why the value

$$r = 0.90$$

provides strong evidence to support the researcher's belief that the transformed model is a more appropriate representation of the data than the original linear model.

(c) Show that the regression equation can be rewritten in the form

$$y = ax^n$$

where a and n are constants to be determined.

(6)

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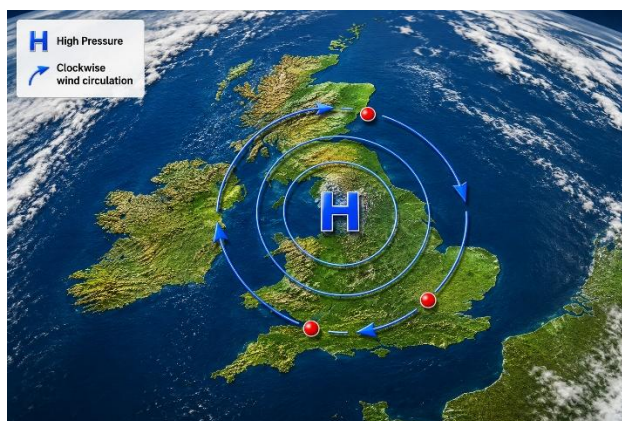
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5.



Mina is studying the large data set for April 2016.

She codes the variable Daily Mean Pressure, x , using

$$y = x - 1010$$

The data for all 30 days from a location are summarised by

$$\sum y = -60 \quad \sum y^2 = 7200$$

- State the units of the variable x .
- Find the mean Daily Mean Pressure for these 30 days.
- Find the standard deviation of Daily Mean Pressure for these 30 days.

Mina knows that, in the UK, winds circulate

- clockwise around a region of high pressure
- anticlockwise around a region of low pressure

On 18/04/2016 the Daily Mean Pressure for 3 locations was:

Location	Heathrow	Hurn	Leuchars
Daily Mean Pressure	1030	1026	1028
Cardinal Wind Direction			

The Cardinal Wind Directions for these 3 locations (in random order) were:

W, E, NE

You may assume that these 3 locations were under a single region of pressure.

- Place each wind direction into the correct location in the table and give a reason.

(8)

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(a) State:

1. a null hypothesis corresponding to no change in the proportion of late submissions;
2. an alternative hypothesis corresponding to the academic manager's belief.

(c) Calculate the actual significance level of the test based on the critical region found in part

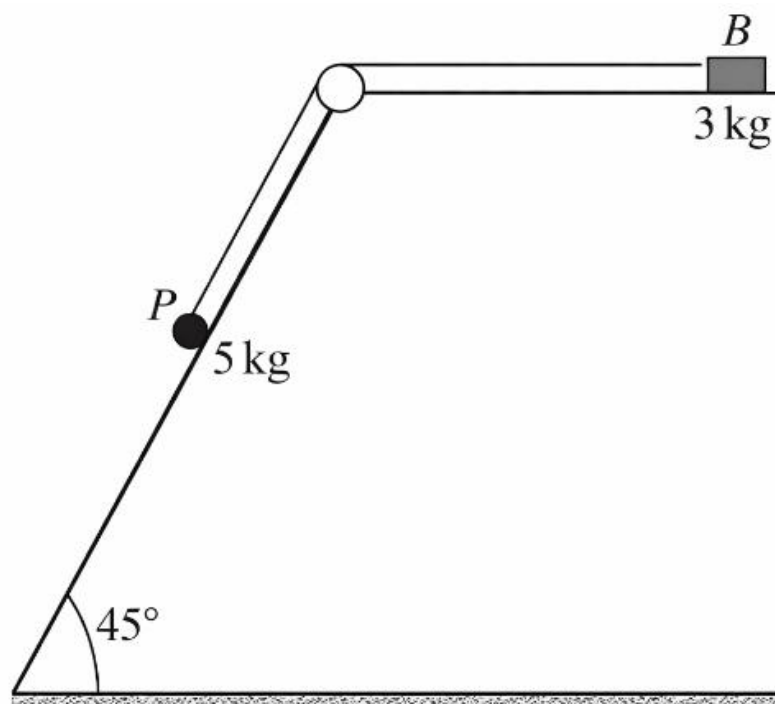
(d). The sample shows that 8 of the 40 students submitted their assignment late. Using your critical region, state whether there is sufficient evidence to support the academic manager's belief. Give a reason for your conclusion.

(7)

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- (4)



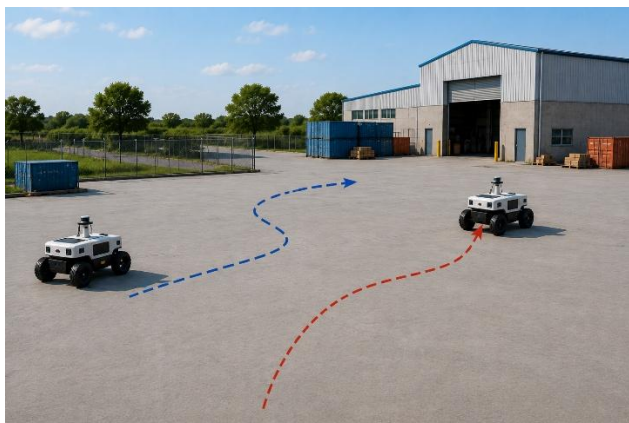
The coefficient of friction between P and the plane is 0.3 and the coefficient of friction between B and the horizontal surface is 0.4. Take $g = 9.8 \text{ m s}^{-2}$.

- (a) Find the tension T in the string (in newtons)
When P is moving at 10 m s^{-1} , the string breaks.
- (b) Find the deceleration of B after the string breaks.

(12)

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10 m s^{-1} .

(9)

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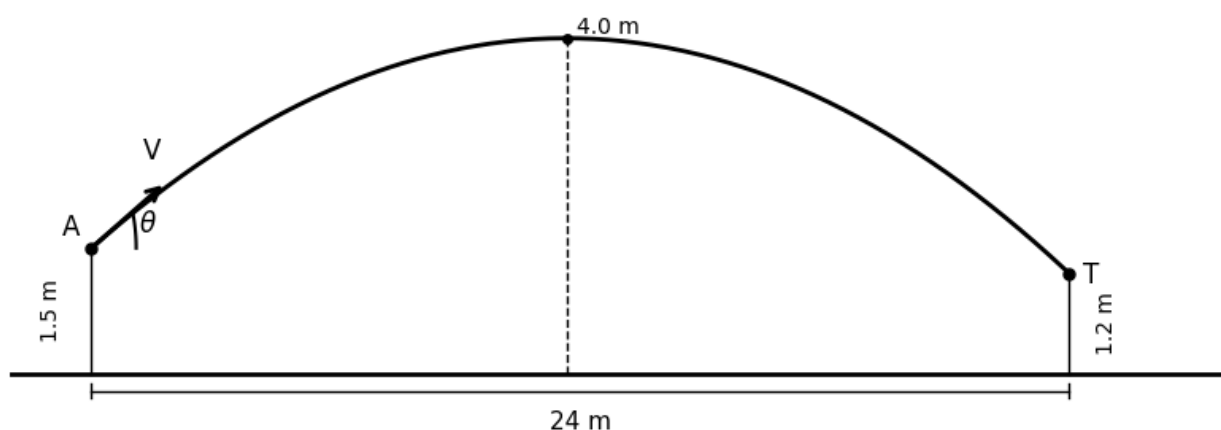
A small blue and white boat is on a calm river in a village. The boat has a white cabin and a blue hull. The river is surrounded by green grass and trees. In the background, there are several houses with red roofs and white walls. The water is still, reflecting the surrounding scenery.



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11.



A student kicks a ball towards a target. When the ball leaves the foot at point A, it is 1.5 m above level ground and is projected with speed V at an angle θ above the horizontal, as in Figure. During its flight, the highest point reached by the ball is 4.0 m above the ground. The target is modelled as a point T, which is 24 m horizontally from A and 1.2 m above the ground.

The ball is modelled as a particle moving freely under gravity $g = 9.8 \text{ m s}^{-2}$.

(a) Show that

$$V^2 = \frac{5g}{\sin^2 \theta}.$$

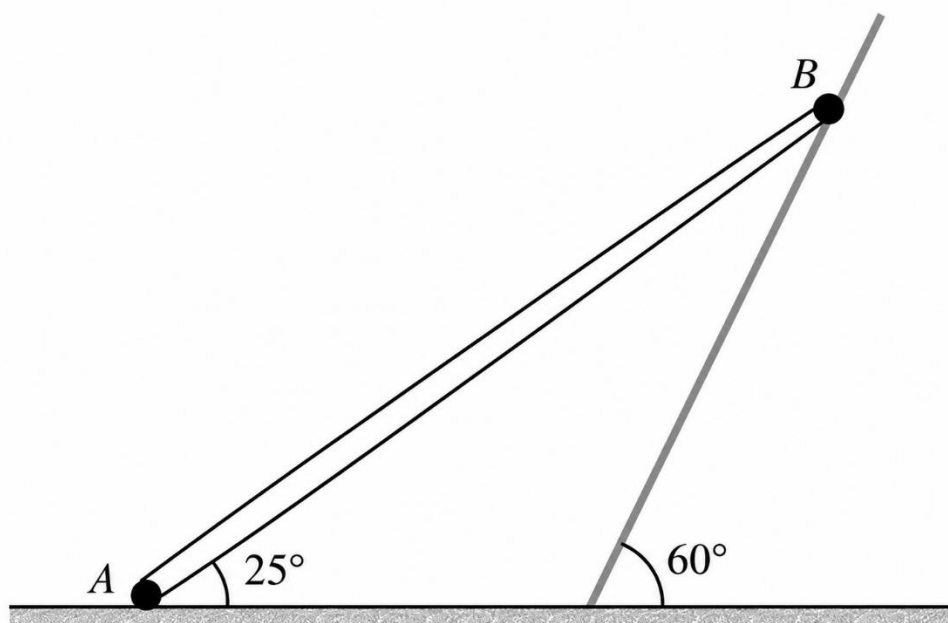
(b) Find the size of θ .

(c) State one limitation of the model that could affect your answer to (b).

(7)

[illegible]

12.



A uniform rod AB of weight 24 N and length 3.0 m rests in equilibrium with end A in contact with rough horizontal ground and end B resting against a smooth wall inclined at 60° to the horizontal. The rod lies in a vertical plane perpendicular to the wall and is inclined at 25° to the horizontal.

- (a) Show that the magnitude of the force acting on the rod at B is 13.3 N
 (b) Determine the magnitude of the contact force between the rod and the ground at A .

(10)

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Total for Paper is 100 Marks

A Level Maths Predicted Paper 2027 (June) Statistics and Mechanics: Paper 3: Set 1: Answers

1. (a) Test for positive correlation

Let ρ represent the population correlation coefficient between GDP per capita and annual CO₂ emissions.

$$H_0: \rho = 0$$

$$H_1: \rho > 0$$

This is a one-tailed test at the 5% significance level.

For a sample of 20 countries, the critical value of the product moment correlation coefficient is approximately

$$r_{\text{critical}} = 0.378.$$

The observed value is

$$r = 0.41.$$

Since

$$0.41 > 0.378,$$

the observed correlation lies in the critical region.

Therefore, we reject H_0 .

There is sufficient evidence at the 5% significance level to conclude that there is a positive correlation between GDP per capita and average annual CO₂ emissions for all countries.

(b) Interpretation of the transformed correlation

For the original variables,

$$r = 0.41,$$

which indicates only a moderate positive linear correlation.

After applying the logarithmic transformations,

$$r = 0.90.$$

This value is much closer to 1, indicating a very strong positive linear relationship between $\log_{10} x$ and $\log_{10} y$.

Therefore, the transformed data lie much closer to a straight line. This supports the researcher's belief that a non-linear power model is more suitable than a linear model for the original variables x and y .

(c) Expressing the relationship as $y = ax^n$

The transformed regression model is

$$c = -2.05 + 0.76m.$$

Given that

$$c = \log_{10} y$$

and

$$m = \log_{10} x,$$

substitute these into the regression equation:

$$\log_{10} y = -2.05 + 0.76 \log_{10} x.$$

$$\log_{10} y = \log_{10}(10^{-2.05}) + \log_{10}(x^{0.76}).$$

$$\log_{10} y = \log_{10}(10^{-2.05} x^{0.76}).$$

$$y = 10^{-2.05}x^{0.76}.$$

$$y = 0.00891x^{0.76}$$

$$y = ax^n,$$

$$a = 0.00891$$

$$n = 0.76$$

2.

$$P(A) = 0.30, P(B) = 0.50, P(C) = 0.20$$

- B and C are independent
- A and C are mutually exclusive $\Rightarrow P(A \cap C) = 0$
- $P(A \cap B) = 0.12$

(a) Venn diagram values (key regions)

First find important intersections:

- $P(B \cap C) = P(B)P(C) = 0.50 \times 0.20 = 0.10$
- $P(A \cap C) = 0$

Now split regions:

- $A \cap B = 0.12$
- $B \cap C = 0.10$
- Triple intersection = 0 (since $A \cap C = 0$)

Only regions:

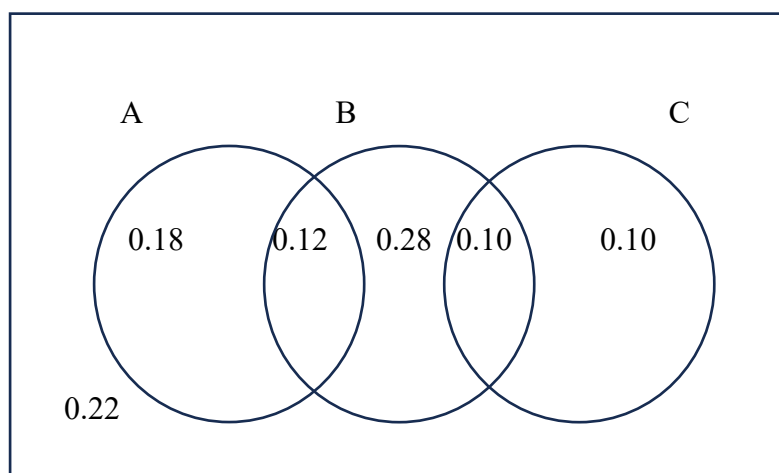
- A only = $0.30 - 0.12 = 0.18$
- B only = $0.50 - 0.12 - 0.10 = 0.28$
- C only = $0.20 - 0.10 = 0.10$

Total inside:

$$0.18 + 0.28 + 0.10 + 0.12 + 0.10 = 0.78$$

Outside (none of A, B, C):

$$1 - 0.78 = 0.22$$



(b)(i) $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.30 + 0.50 - 0.12 = 0.68$$

(ii) $P(A' \cap C')$

$$P(A' \cap C') = 1 - P(A \cup C)$$

$$P(A \cup C) = P(A) + P(C) - P(A \cap C) = 0.30 + 0.20 - 0 = 0.50$$

$$P(A' \cap C') = 1 - 0.50 = 0.50$$

(iii) $P(A | B)$

$$P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.50} = 0.24$$

(iv) $P(B | A' \cap C')$

$$P(B | A' \cap C') = \frac{P(B \cap A' \cap C')}{P(A' \cap C')}$$

From the diagram:

- $B \cap A' \cap C' = B \text{ only} = 0.28$
- $P(A' \cap C') = 0.50$

$$= \frac{0.28}{0.50} = 0.56$$

3. Let $X \sim N(168, 6.5^2)$.

(a) Find k given $P(X < k) = 0.02$

$$P\left(Z < \frac{k - 168}{6.5}\right) = 0.02 \Rightarrow \frac{k - 168}{6.5} = z_{0.02} \approx -2.054$$

$$k = 168 + (-2.054)(6.5) \approx 168 - 13.35 = 154.65$$

$$\boxed{k \approx 154.7 \text{ cm}}$$

(b) Proportion between 155 cm and 180 cm

$$P(155 < X < 180) = \Phi\left(\frac{180 - 168}{6.5}\right) - \Phi\left(\frac{155 - 168}{6.5}\right)$$

$$= \Phi(1.846) - \Phi(-2.000) \approx 0.9676 - 0.0228 = 0.9448$$

$$\boxed{0.945 \text{ (approximately)}}$$

(c) Conditional probability $P(X > 165 | 155 < X < 180)$

$$P(X > 165 | 155 < X < 180) = \frac{P(165 < X < 180)}{P(155 < X < 180)}$$

Denominator from (b): ≈ 0.9448

Numerator:

$$P(165 < X < 180) = \Phi(1.846) - \Phi\left(\frac{165 - 168}{6.5}\right)$$

$$\frac{165 - 168}{6.5} = -0.462 \quad \Phi(-0.462) \approx 0.3222$$

$$\Rightarrow 0.9676 - 0.3222 = 0.6454$$

$$\text{So } \frac{0.6454}{0.9448} \approx 0.683$$

$$\boxed{0.683 \text{ (approximately)}}$$

(d) Hypothesis test (5% level)

Given $\sigma = 7.8$, $n = 60$, $\bar{x} = 166.9$, test mean < 168 .

Hypotheses

$$H_0: \mu = 168 \quad H_1: \mu < 168$$

Test statistic (z-test, σ known):

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{166.9 - 168}{7.8/\sqrt{60}}$$

$$7.8/\sqrt{60} \approx 7.8/7.746 \approx 1.007$$

$$z \approx \frac{-1.1}{1.007} \approx -1.09$$

Critical value for 5% one-tailed: -1.645 .

Since $-1.09 > -1.645$, we do not reject H_0 .

(Equivalently, $p\text{-value} \approx \Phi(-1.09) \approx 0.14 > 0.05$.)

4. (a)

$$P(X = r) = P(X = r + 4) \quad r = 1, 2, 3, 4.$$

$$P(X = 5) = P(X = 1) = 0.08$$

$$P(X = 6) = P(X = 2) = 0.14$$

$$P(X = 7) = P(X = 3) = 0.18$$

$$P(X = 4) = P(X = 8) = p.$$

The probabilities must add up to 1, so

$$0.08 + 0.14 + 0.18 + p + 0.08 + 0.14 + 0.18 + p = 1.$$

$$0.80 + 2p = 1.$$

$$p = 0.10.$$

$$P(X = x) = \begin{cases} 0.08, & x = 1, 5, \\ 0.14, & x = 2, 6, \\ 0.18, & x = 3, 7, \\ 0.10, & x = 4, 8. \end{cases}$$

(b) Let W be the number of times that the wheel lands on 7 in 30 independent spins.

$$P(X = 7) = 0.18,$$

$$W \sim B(30, 0.18).$$

$$P(W \geq 8).$$

$$P(W \geq 8) = 1 - P(W \leq 7).$$

$$P(W \geq 8) = 1 - \sum_{w=0}^7 \binom{30}{w} (0.18)^w (0.82)^{30-w}.$$

Using a calculator,

$$P(W \geq 8) = 0.1582117 \dots$$

$$\boxed{P(W \geq 8) = 0.158}.$$

(c)

$$Y = \frac{28}{X},$$

$$P(Y - X \leq 3).$$

$$\frac{28}{X} - X \leq 3.$$

$$X^2 + 3X - 28 \geq 0.$$

$$(X + 7)(X - 4) \geq 0.$$

$$X \leq -7 \text{ or } X \geq 4.$$

$$X = 4, 5, 6, 7, 8.$$

$$\begin{aligned} P(Y - X \leq 3) &= P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8) \\ &= 0.10 + 0.08 + 0.14 + 0.18 + 0.10 \\ &= 0.60. \end{aligned}$$

$$P(Y - X \leq 3) = 0.60.$$

5. (a) Units: hectopascals (hPa) (equivalently millibars, mb).

(b) Mean Daily Mean Pressure

$$\bar{y} = \frac{\sum y}{n} = \frac{-60}{30} = -2$$

Since $y = x - 1010$, then $x = y + 1010$, so

$$\bar{x} = \bar{y} + 1010 = -2 + 1010 = 1008$$

Mean Daily Mean Pressure = **1008 hPa**

(c) Standard deviation

$$\begin{aligned} s &= \sqrt{\frac{\sum y^2}{n} - \bar{y}^2} \\ s &= \sqrt{\frac{7200}{30} - (-2)^2} = \sqrt{240 - 4} = \sqrt{236} \approx 15.36 \end{aligned}$$

Standard deviation of x is the same as standard deviation of y (subtracting 1010 shifts data but doesn't change spread).

$$s \approx 15.4 \text{ hPa}$$

(d) Wind directions in the table

Since Heathrow has the highest pressure (1030 hPa), we assume the centre of a high is closest to Heathrow. Winds around a high pressure system in the UK are clockwise.

With Heathrow roughly between Leuchars (north) and Hurn (south):

- North of a high $\rightarrow$ wind tends to be from the East (E)
- South of a high $\rightarrow$ wind tends to be from the West (W)

Location	Heathrow	Hurn	Leuchars
Cardinal Wind Direction	NE	W	E

Reason (example wording): Heathrow has the highest pressure so is nearest the high-pressure centre; winds circulate clockwise around a high, giving E to the north (Leuchars) and W to the south (Hurn), leaving NE for Heathrow.

6. (a) Let p be the proportion of students who submit assignments late.

The null hypothesis is

$$H_0: p = 0.12$$

The manager believes that the proportion has changed, so the alternative hypothesis is two-tailed:

$$H_1: p \neq 0.12$$

(b) Under the null hypothesis,

$$X \sim B(40, 0.12).$$

For a two-tailed test at the 5% significance level, each tail should have probability less than

$$\frac{0.05}{2} = 0.025.$$

Lower tail

Consider

$$P(X \leq 0) = P(X = 0).$$

$$P(X = 0) = (0.88)^{40}.$$

$$P(X \leq 0) \approx 0.006016.$$

This is less than 0.025.

However,

$$P(X \leq 1) \approx 0.03883,$$

which is greater than 0.025.

Therefore, the lower critical region is

$$X = 0.$$

Upper tail

We seek the smallest value k such that

$$P(X \geq k) < 0.025.$$

Using binomial probabilities,

$$P(X \geq 9) \approx 0.04431,$$

which is greater than 0.025.

But

$$P(X \geq 10) \approx 0.01749,$$

which is less than 0.025.

Therefore, the upper critical region is

$$X \geq 10.$$

Hence the complete critical region is

$$X = 0 \text{ or } X \geq 10.$$

The tail probabilities are

$$P(X \leq 0) \approx 0.006016$$

and

$$P(X \geq 10) \approx 0.01749.$$

Both are less than 0.025.

(c) The actual significance level is the total probability of the critical region:

$$P(X = 0) + P(X \geq 10).$$

$$= 0.00601599 + 0.01749078.$$

$$= 0.02350677.$$

Therefore, the actual significance level is

$$0.0235$$

$$2.35\%.$$

(d) The observed value is

$$X = 8.$$

The critical region is

$$X = 0 \text{ or } X \geq 10.$$

Since

does not lie in the critical region, we do not reject the null hypothesis.

Therefore, there is insufficient evidence at the 5% significance level to conclude that the proportion of students submitting assignments late has changed.

There is insufficient evidence to support the manager's belief.

7.

$$u = 0 \quad a = 2.5 \quad t = 6$$

(a)

$$v = u + at = 0 + 2.5(6) = 15$$

$$v = 15 \text{ m s}^{-1}$$

(b)

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2}(2.5)(6^2) = 1.25(36) = 45$$

$$s = 45 \text{ m}$$

8.

- $m_B = 3 \text{ kg}$, on rough horizontal, $\mu_B = 0.4$
- $m_P = 5 \text{ kg}$, on rough plane at 45° , $\mu_P = 0.3$
- $g = 9.8 \text{ m s}^{-2}$
- While string is intact: P moves down the plane, B moves towards the pulley.

(a) Tension T in the string

Let the common acceleration be a (down the plane for P , towards pulley for B).

For P (down the plane positive)

Normal reaction:

$$R_P = 5g \cos 45^\circ$$

Friction (up the plane):

$$F_P = \mu_P R_P = 0.3(5g \cos 45^\circ)$$

Equation along plane:

$$5a = 5g \sin 45^\circ - T - 0.3(5g \cos 45^\circ)$$

Since $\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$:

$$5a = \frac{5g}{\sqrt{2}} - T - \frac{0.3 \cdot 5g}{\sqrt{2}} = \frac{5g(1 - 0.3)}{\sqrt{2}} - T = \frac{3.5g}{\sqrt{2}} - T$$

$$5a = \frac{3.5g}{\sqrt{2}} - T$$

For B (towards pulley positive)

Normal reaction = $3g$. Friction opposing motion:

$$F_B = \mu_B(3g) = 0.4(3g) = 1.2g$$

Equation horizontally:

$$3a = T - 1.2g$$

$$T = 3a + 1.2g$$

$$5a = \frac{3.5g}{\sqrt{2}} - (3a + 1.2g)$$

$$a = 1.5625$$

$$T = 3a + 1.2g$$

$$\boxed{T \approx 16.4 \text{ N}}$$

(b) Deceleration of B after the string breaks

After the string breaks, B is no longer pulled by tension, so the only horizontal force is friction:

$$F_B = \mu_B(3g) = 1.2g$$

This acts opposite to the motion, so the deceleration magnitude is

$$a_B = \frac{F_B}{m_B} = \frac{1.2g}{3} = 0.4g$$

$$a_B = 0.4(9.8) = 3.92 \text{ ms}^{-2}$$

9. (a)

$$a = \frac{dv}{dt}$$

$$\frac{dv}{dt} = (2 - 6t)\mathbf{i} + (4 - t^2)\mathbf{j}.$$

Integrating each component gives

$$\mathbf{v} = (2t - 3t^2 + C_1)\mathbf{i} + \left(4t - \frac{t^3}{3} + C_2\right)\mathbf{j}.$$

When $t = 0$,

$$\mathbf{v} = 5\mathbf{i} - 2\mathbf{j}.$$

Therefore,

$$C_1 = 5 \text{ and } C_2 = -2.$$

Hence, the velocity at time t is

$$\boxed{\mathbf{v} = (5 + 2t - 3t^2)\mathbf{i} + \left(-2 + 4t - \frac{t^3}{3}\right)\mathbf{j}}.$$

When $t = 3$, the $\mathbf{i}$ -component is

$$5 + 2(3) - 3(3^2) = 5 + 6 - 27 = -16.$$

The $\mathbf{j}$ -component is

$$-2 + 4(3) - \frac{3^3}{3} = -2 + 12 - 9 = 1.$$

$$\boxed{\mathbf{v} = -16\mathbf{i} + \mathbf{j} \text{ m s}^{-1}}.$$

(b) Motion perpendicular to $\mathbf{i}$

A particle moving perpendicular to $\mathbf{i}$ has no velocity component in the $\mathbf{i}$ -direction.

Therefore, set the $\mathbf{i}$ -component of velocity equal to zero:

$$5 + 2t - 3t^2 = 0.$$

$$3t^2 - 2t - 5 = 0.$$

$$(3t - 5)(t + 1) = 0.$$

Since $t \geq 0$

$$\boxed{t = \frac{5}{3} \text{ seconds}}.$$

(c) Time when the speed of Q is 10 m s^{-1}

The position vector is

$$\mathbf{r} = (t^2 + 2t)\mathbf{i} + 4t\mathbf{j}.$$

Differentiate the position vector to obtain the velocity:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}.$$

$$\mathbf{v} = (2t + 2)\mathbf{i} + 4\mathbf{j}.$$

The speed is the magnitude of the velocity vector:

$$|\mathbf{v}| = \sqrt{(2t+2)^2 + 4^2}.$$

$$\sqrt{(2t+2)^2 + 16} = 10.$$

$$(2t + 2)^2 + 16 = 100.$$

$$(2t + 2)^2 = 84.$$

$$2t + 2 = \pm\sqrt{84}.$$

$$t = -1 \pm \sqrt{21}.$$

Since $t \geq 0$

$$t = \sqrt{21} - 1 \text{ seconds}.$$

$$t \approx 3.58 \text{ seconds}.$$

10. (a) The resultant force is

$$\mathbf{F} = (6 + \lambda)\mathbf{i} + (2 + \mu)\mathbf{j}.$$

Since the ship moves in the direction $2\mathbf{i} - \mathbf{j}$, the resultant force must be parallel to this vector. Therefore,

$$(6 + \lambda)\mathbf{i} + (2 + \mu)\mathbf{j} = k(2\mathbf{i} - \mathbf{j}).$$

Comparing components:

$$6 + \lambda = 2k,$$

$$2 + \mu = -k.$$

$$k = -(2 + \mu).$$

Substituting:

$$6 + \lambda = -2(2 + \mu),$$

$$6 + \lambda = -4 - 2\mu.$$

$$\lambda + 2\mu + 10 = 0.$$

(b) When $\lambda = 0$,

$$2\mu + 10 = 0,$$

$$\mu = -5.$$

The resultant force is therefore

$$\mathbf{F} = 6\mathbf{i} - 3\mathbf{j}.$$

Using $\mathbf{F} = m\mathbf{a}$,

$$\mathbf{a} = \frac{1}{5}(6\mathbf{i} - 3\mathbf{j}).$$

The magnitude of the acceleration is

$$|a| = \sqrt{\left(\frac{6}{5}\right)^2 + \left(-\frac{3}{5}\right)^2} = \frac{3\sqrt{5}}{5} \text{ m s}^{-2}.$$

Since the toy ship starts from rest,

$$s = ut + \frac{1}{2}at^2.$$

With $u = 0$ and $t = 3$,

$$AB = \frac{1}{2} \left(\frac{3\sqrt{5}}{5} \right) (3^2).$$

$$AB = \frac{27\sqrt{5}}{10} \text{ m}$$

$$AB \approx 6.04 \text{ m}.$$

11. (a) Vertical rise from A to the highest point:

$$4.0 - 1.5 = 2.5 \text{ m}$$

At the highest point, vertical velocity = 0. Using

$$v^2 = u^2 + 2as$$

in the vertical direction:

$$0 = (V \sin \theta)^2 - 2g(2.5)$$

$$(V \sin \theta)^2 = 5g$$

$$V^2 = \frac{5g}{\sin^2 \theta}$$

(b) Horizontal motion to find t

Horizontal acceleration is 0, so:

$$x = (V \cos \theta)t$$

Given $x = 24$:

$$24 = (V \cos \theta)t$$

$$t = \frac{24}{V \cos \theta}$$

Vertical motion using $s = ut + \frac{1}{2}at^2$

Vertical displacement from A to T:

$$s = 1.2 - 1.5 = -0.3$$

Vertical initial velocity:

$$u = V \sin \theta = 7$$

Vertical acceleration:

$$a = -g = -9.8$$

$$s = ut + \frac{1}{2}at^2$$

$$-0.3 = 7t + \frac{1}{2}(-9.8)t^2$$

$$-0.3 = 7t - 4.9t^2$$

$$49t^2 - 70t - 3 = 0$$

$$t = \frac{70 \pm \sqrt{70^2 + 4(49)(3)}}{2 \cdot 49} = \frac{70 \pm \sqrt{4900 + 588}}{98} = \frac{70 \pm \sqrt{5488}}{98}$$

$$t = \frac{70 \pm 28\sqrt{7}}{98} = \frac{5 \pm 2\sqrt{7}}{7}$$

Time must be positive, so:

$$t = \frac{5 + 2\sqrt{7}}{7}$$

Use horizontal motion again to get $\tan \theta$

From horizontal:

$$24 = (V \cos \theta)t \quad t \Rightarrow V \cos \theta = \frac{24}{t}$$

$$\tan \theta = \frac{V \sin \theta}{V \cos \theta}$$

But $V \sin \theta = 7$, so

$$\tan \theta = \frac{7}{24/t} = \frac{7t}{24}$$

Substitute $t = \frac{5+2\sqrt{7}}{7}$:

$$\tan \theta = \frac{7}{24} \cdot \frac{5 + 2\sqrt{7}}{7} = \frac{5 + 2\sqrt{7}}{24}$$

$$\tan \theta = \frac{5 + 2\sqrt{7}}{24}$$

$$\theta = \tan^{-1} \left(\frac{5 + 2\sqrt{7}}{24} \right) \approx 23.2^\circ$$

(c)

Air resistance is ignored (it would change the path and angle needed).

12.

(a) Show $R_B = 13.3 \text{ N}$

Take moments about A.

- Moment of W about A: perpendicular distance is the horizontal distance of the midpoint:

$$\left(\frac{L}{2}\right) \cos 25^\circ = 1.5 \cos 25^\circ$$

Clockwise moment:

$$24(1.5 \cos 25^\circ)$$

Anticlockwise moment:

$$R_B(3 \sin 55^\circ)$$

Equilibrium:

$$R_B(3 \sin 55^\circ) = 24(1.5 \cos 25^\circ)$$

$$R_B = \frac{12 \cos 25^\circ}{\sin 55^\circ}$$

$$13.28 \approx \boxed{13.3 \text{ N}}$$

(b) Contact force between rod and ground at A

Resolve forces.

Components of R_B

$$R_{Bx} = R_B \cos 30^\circ = 13.28(0.8660) = 11.50$$

$$R_{By} = R_B \sin 30^\circ = 13.28(0.5) = 6.64$$

Horizontal equilibrium

$$F = R_{Bx} \Rightarrow F = 11.50 \text{ N}$$

Vertical equilibrium

$$N + R_{By} = 24 \Rightarrow N = 24 - 6.64 = 17.36 \text{ N}$$

Resultant contact force at A

$$R_A = \sqrt{N^2 + F^2} = \sqrt{(17.36)^2 + (11.50)^2}$$

$$R_A = \sqrt{301.4 + 132.3} = \sqrt{433.7} = 20.82 \approx \boxed{20.8 \text{ N}}$$

END